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Boltzmann-Gibbs entropy is sufficient but not necessary for the likelihood factorization required by Einstein

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IOPscience

Highlights-of-2015

(21 March 2016: 624 downloads)

Einstein 1910 (reversal of Boltzmann formula):

For any two independent systems A and B,

the likelihood function should satisfy

$$\Omega(A+B)=\Omega(A)\;\Omega(B)$$ (Einstein principle)

$q=1:$ $S_{BG}=k_B \ln W$ hence $\Omega(\{p_i\}) \propto e^{S_{BG}(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto e^{S_{BG}(A+B)/k_B} = e^{S_{BG}(A)/k_B + S_{BG}(B)/k_B} = e^{S_{BG}(A)/k_B} e^{S_{BG}(B)/k_B} \propto \Omega(A)\;\Omega(B)$$

OK!

$\forall q:$ $S_q = k_B \ln_q W$ hence $\Omega(\{p_i\}) \propto e_q^{S_q(\{p_i\})/k_B}$ hence

$$\Omega(A+B) \propto \mathrm{e}_q^{S_q(A+B)/k_B} = \mathrm{e}_q^{S_q(A)/k_B + S_q(B)/k_B + (1-q)[S_q(A)/k_B][S_q(B)/k_B]}$$

$$= e_q^{S_q(A)/k_B} e_q^{S_q(B)/k_B} \propto \Omega(A)\;\Omega(B)$$

OK $\forall q$!

COMPOSITION OF VELOCITIES OF INERTIAL SYSTEMS (d=1)

$$v_{13} = v_{12} + v_{23} \qquad \text{(Galileo)}$$

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \qquad \text{(Einstein)}$$

Newton mechanics:

It satisfies Galilean additivity **but** violates Lorentz invariance (hence mechanics can not be unified with Maxwell electromagnetism)

Einstein mechanics (Special relativity):

It satisfies Lorentz invariance (hence mechanics is unified with Maxwell electromagnetism) **but** violates Galilean additivity

Question: which is physically more fundamental, the additive composition of velocities **or** the unification of mechanics and electromagnetism?

Special relativity recovers Newtonian/Galilean mechanics as particular case:

$$v_{13} = \frac{v_{12} + v_{23}}{1 + \frac{v_{12}}{c}\frac{v_{13}}{c}} \sim v_{12} + v_{23}$$

if $1/c \to 0$ or $\forall\ 1/\mathrm{c} \neq 0$ with $v/c \to 0$

q-statistics recovers Boltzmann-Gibbs statistics as particular case:

$$\mathrm{e}_q^{-\beta E} \equiv \frac{1}{\left[1 + (q-1)\beta E\right]^{\frac{1}{q-1}}} \sim e^{-\beta E}$$

if $(q\text{-}1) \to 0$ or $\forall\ (\mathrm{q}\text{-}1) \neq 0$ with $\beta \mathrm{E} \to 0$

TRIANGLE FOR INDEPENDENT COINS

(N=0) $1 \times \frac{1}{1}$

(N=1) $1 \times \frac{1}{2}$ $1 \times \frac{1}{2}$

(N=2) $1 \times \frac{1}{4}$ $2 \times \frac{1}{4}$ $1 \times \frac{1}{4}$

(N=3) $1 \times \frac{1}{8}$ $3 \times \frac{1}{8}$ $3 \times \frac{1}{8}$ $1 \times \frac{1}{8}$

(N=4) $1 \times \frac{1}{16}$ $4 \times \frac{1}{16}$ $6 \times \frac{1}{16}$ $4 \times \frac{1}{16}$ $1 \times \frac{1}{16}$

(N=5) $1 \times \frac{1}{32}$ $5 \times \frac{1}{32}$ $10 \times \frac{1}{32}$ $10 \times \frac{1}{32}$ $5 \times \frac{1}{32}$ $1 \times \frac{1}{32}$

$$q_{entropy} = 1$$

$$q_{limit} = 1$$

$$\sum_{n=0}^{N} \binom{N}{n} r_{N,n} = 1 \quad (\forall\ N)$$

HYBRID PASCAL - LEIBNITZ TRIANGLE

(N=0) $1\times\frac{1}{1}$

(N=1) $1\times\frac{1}{2}$ $1\times\frac{1}{2}$

(N=2) $1\times\frac{1}{3}$ $2\times\frac{1}{6}$ $1\times\frac{1}{3}$

(N=3) $1\times\frac{1}{4}$ $3\times\frac{1}{12}$ $3\times\frac{1}{12}$ $1\times\frac{1}{4}$

(N=4) $1\times\frac{1}{5}$ $4\times\frac{1}{20}$ $6\times\frac{1}{30}$ $4\times\frac{1}{20}$ $1\times\frac{1}{5}$

(N=5) $1\times\frac{1}{6}$ $5\times\frac{1}{30}$ $10\times\frac{1}{60}$ $10\times\frac{1}{60}$ $5\times\frac{1}{30}$ $1\times\frac{1}{6}$

$$\sum_{n=0}^{N}\binom{N}{n} r_{N,n} = 1 \quad (\forall N)$$

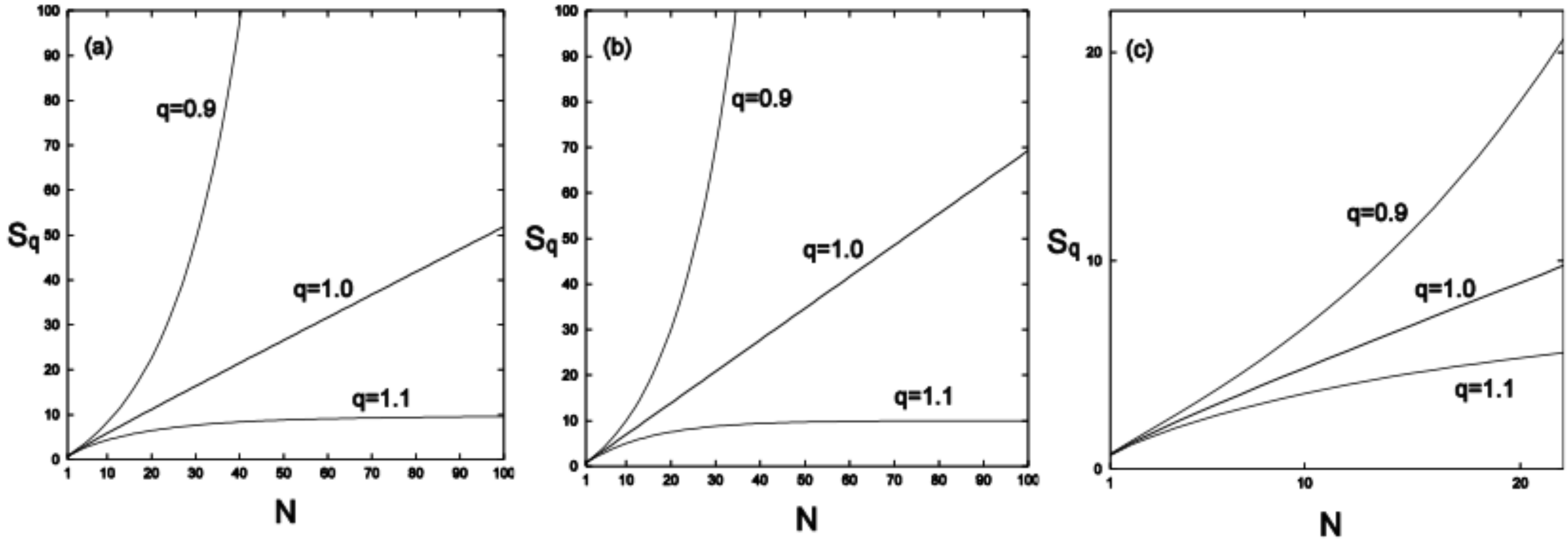
Blaise **Pascal** (1623-1662)
Gottfried Wilhelm **Leibnitz** (1646-1716)
Daniel **Bernoulli** (1700-1782)

$q = 1$ SYSTEMS

i.e., such that $S_1(N) \propto N \quad (N \to \infty)$

I don't believe that atoms exist!

Ernst Mach (January 1897, Vienna)



Leibnitz triangle

$$\left(p_{N,0} = \frac{1}{N+1} \right)$$

N independent coins

$$\left(\begin{array}{l} p_{N,0} = p^N \\ \\ \text{with } p = 1/2 \end{array} \right)$$

Stretched exponential

$$\left(\begin{array}{l} p_{N,0} = p^{N^\alpha} \\ \\ \text{with } p = \alpha = 1/2 \end{array} \right)$$

(All three examples strictly satisfy the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA 102, 15377 (2005)

Asymptotically scale-invariant (d=2)

$(N=0)$	1				
$(N=1)$	$1/2$	$1/2$			
$(N=2)$	$1/3$	$1/6$	$1/3$		
$(N=3)$	$3/8$	$5/48$	$5/48$	0	
$(N=4)$	$2/5$	$3/40$	$1/20$	0	0

d+1

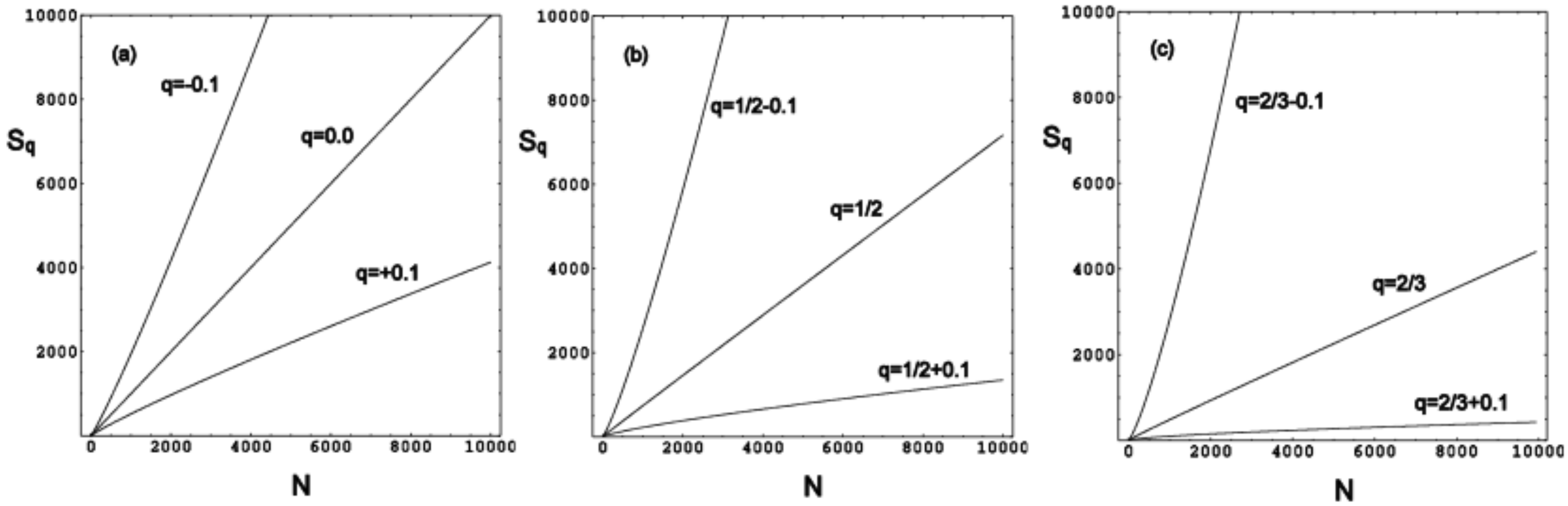
(It asymptotically satisfies the Leibnitz rule)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

$q \neq 1$ SYSTEMS

i.e., such that $S_q(N) \propto N \quad (N \to \infty)$

$(d=1)$ $(d=2)$ $(d=3)$



$$q = 1 - \frac{1}{d}$$

(All three examples **asymptotically** satisfy the **Leibnitz rule**)

C.T., M. Gell-Mann and Y. Sato, Proc Natl Acad Sc USA **102**, 15377 (2005)

An entropy S functional is *additive* if (and only if),

for any two probabilistically independent systems A and B,

$$S(A+B)=S(A)+S(B)$$

i.e., if $p_{i,j}^{A+B}=p_i^A p_j^B \quad (i=1,2,\ldots,W_A;\ j=1,2,\ldots,W_B)$, then

$$S\left(\left\{p_{i,j}^{A+B}\right\}\right)=S\left(\left\{p_i^A\right\}\right)+S\left(\left\{p_j^B\right\}\right)$$

Property of S_q:

$$\sum_{i,j}^{W_A W_B}\left(p_{i,j}^{A+B}\right)^q=\sum_{i,j}^{W_A W_B}\left(p_i^A\right)^q\left(p_j^B\right)^q=\left[\sum_{i=1}^{W_A}\left(p_i^A\right)^q\right]\left[\sum_{j=1}^{W_B}\left(p_j^B\right)^q\right]$$

$$\Rightarrow 1+(1-q)\frac{S_q(A+B)}{k}=\left[1+(1-q)\frac{S_q(A)}{k}\right]\left[1+(1-q)\frac{S_q(B)}{k}\right]$$

$$\Rightarrow \quad \frac{S_q(A+B)}{k}=\frac{S_q(A)}{k}+\frac{S_q(B)}{k}+(1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}$$

i.e., $S_q(A+B)=S_q(A)+S_q(B)+\frac{1-q}{k}S_q(A)S_q(B)$

therefore S_q is nonadditive unless $(1-q)/k\rightarrow 0$

GENERALIZATION: A and B either independent or correlated $\Rightarrow$

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B|A)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B|A)}{k}$$

$$= \frac{S_q(B)}{k} + \frac{S_q(A|B)}{k} + (1-q)\frac{S_q(B)}{k}\frac{S_q(A|B)}{k} \quad (\forall q)$$

Entropy of (A+B) (calculated with **joint** probabilities)

[S. Abe, Phys Lett A 271 (2000) 74]

Entropy of B (calculated with **marginal** probabilities)

Conditional entropy of A (calculated with **conditional** probabilities)

For a special class of correlations, a value of q (noted q_{ent}) exists such as

$$S_{q_{ent}}(A+B) = S_{q_{ent}}(A) + S_{q_{ent}}(B) \qquad \text{(extensivity)}$$

Independence implies:

$$S_q(A|B) = S_q(A) \quad (\forall q)$$

$$S_q(B|A) = S_q(B) \quad (\forall q)$$

$$q_{ent} = 1$$

SANTOS THEOREM: RJV Santos, J Math Phys **38**, 4104 (1997)

(q -generalization of Shannon 1948 theorem)

IF $S(\{p_i\})$ *continuous function of* $\{p_i\}$

AND $S(p_i = 1/W, \forall i)$ *monotonically increases with* W

AND $\frac{S(A+B)}{k} = \frac{S(A)}{k} + \frac{S(B)}{k} + (1-q)\frac{S(A)}{k}\frac{S(B)}{k}$ $(\text{with } p_{ij}^{A+B} = p_i^A p_j^B)$

AND $S(\{p_i\}) = S(p_L, p_M) + p_L^q \, S(\{p_l / p_L\}) + p_M^q \, S(\{p_m / p_M\})$ $(\text{with } p_L + p_M = 1)$

THEN AND ONLY THEN

$$S(\{p_i\}) = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad \left(q = 1 \Rightarrow S(\{p_i\}) = -k\sum_{i=1}^{W} p_i \ln p_i \right)$$

CE SHANNON (The Mathematical Theory of Communication):

"This theorem, and the assumptions required for its proof, are in no way necessary for the present theory. It is given chiefly to lend a certain plausibility to some of our later definitions. The real justification of these definitions, however, will reside in their implications.

ABE THEOREM: S Abe, Phys Lett A 271, 74 (2000)

(q -generalization of Khinchin 1953 theorem)

IF $S(\{p_i\})$ *continuous function of* $\{p_i\}$

AND $S(p_i = 1/W,\ \forall i)$ *monotonically increases with* W

AND $S(p_1, p_2, ..., p_W, 0) = S(p_1, p_2, ..., p_W)$

AND $$\frac{S(A+B)}{k} = \frac{S(A)}{k} + \frac{S(B|A)}{k} + (1-q)\frac{S(A)}{k}\frac{S(B|A)}{k}$$

where $S(A+B) \equiv S\left(\{p_{ij}^{A+B}\}\right)$

$$S(A) \equiv S\left(\left\{\sum_{j=1}^{W_B} p_{ij}^{A+B}\right\}\right)$$

$$S(B|A) \equiv \frac{\sum_{i=1}^{W_A} (p_i^A)^q\, S\left(\left\{\frac{p_{ij}^{A+B}}{p_i^A}\right\}\right)}{\sum_{i=1}^{W_A} (p_i^A)^q}$$

THEN AND ONLY THEN

$$S(\{p_i\}) = k\frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad \left(q = 1 \Rightarrow S(\{p_i\}) = -k\sum_{i=1}^{W} p_i \ln p_i\right)$$

The possibility of such theorem was conjectured by AR Plastino and A Plastino (1996, 1999).

Shore and Johnson axioms (1980)

(1) Uniqueness says that the function $H(\{p_k\})$ must be convex, so that there will only be a single maximum, i.e., a single set of values $\{p_k^*\}$.

(2) Coordinate system invariance says that predictions made from an inference should be independent of the choice of coordinate system. It is relevant when the probabilities are continuous functions and determining the dependence of H on the prior over p_k.

(3) Subset independence says that if probability p_k of bin k increases by δp and the probability p_j of bin j correspondingly decreases by δp, then no other bins are affected by the change. Subset independence yields the relationship

$$H = \sum_k f(p_k) + C,$$

where C is a constant independent of p_k.

(4) System independence says that bringing together two systems having probabilities $\mathbf{u} = \{u_i\}$ and $\mathbf{v} = \{v_j\}$ gives new bins that have probability $\mathbf{p} = \mathbf{u} \times \mathbf{v}$, where $p_{ij} = u_i v_j$. The systems are considered independent if constraints on the data do not couple them.

Presse et al, PRL **111**, 180604 (2013)

PRL 111, 180604 (2013) PHYSICAL REVIEW LETTERS week ending 1 NOVEMBER 2013

Nonadditive Entropies Yield Probability Distributions with Biases not Warranted by the Data

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Different quantities that go by the name of entropy are used in variational principles to infer probability distributions from limited data. Shore and Johnson showed that maximizing the Boltzmann-Gibbs form of the entropy ensures that probability distributions inferred satisfy the multiplication rule of probability for independent events in the absence of data coupling such events. Other types of entropies that violate the Shore and Johnson axioms, including nonadditive entropies such as the Tsallis entropy, violate this basic consistency requirement. Here we use the axiomatic framework of Shore and Johnson to show how such nonadditive entropy functions generate biases in probability distributions that are not warranted by the underlying data.

Shore and Johnson axioms 1980 mandate Boltzmann-Gibbs-Shannon entropy…
hence they need to be generalized!!!

Compromise of mathematics is with logics. Only when nature 'likes it', mathematics becomes theoretical physics!

Entropy **2015**, *17*, 2853-2861; doi:10.3390/e17052853

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Article

Conceptual Inadequacy of the Shore and Johnson Axioms for Wide Classes of Complex Systems

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Euclid set of axioms *including his celebrated 5th postulate* yields the magnificent Euclidean geometry

Violation of the 5th postulate yields Riemannian geometries

Carl Friedrich Gauss 1813
Ferdinand Karl Schweikart 1818
János Bolyai 1830
Nikolai Ivanovich Lobachevsky 1830
Bernhard Riemann 1854

If we stubbornly insisted that the 5th postulate was not proposed by Euclid but was mandated by God, then General Relativity would not exist! ☹

PREDECESSORS

RENYI ENTROPY $\propto \ln \sum_i p_i^q$:

M.P. Schutzenberger, *Publ. Inst. Statist. Univ. Paris* (1954) [*according to* I. Csiszar (1974,1978)]

A. Renyi, *Proc.* 4th *Berkeley Symposium* (1969)

ENTROPY $\propto 1-\sum_i p_i^q$:

J. Harvda and F. Charvat, *Kybernetica* 3, 30 (1967)

I. Vajda, *Kybernetica* 4, 105 (1968)

Z. Daroczy, *Inf. Control* 16, 36 (1970)

J. Lindhard and V. Nielsen, *Det Kongelige Danske Videnskabernes Selskab Matematisk - fysiske Meddelelser* (*Denmark*) 38 (9), 1 (1971)

A.M. Mathai and P.N. Rathie, *Basic Concepts in Information Theory and Statistics: Axiomatic Foundations and Applications* (*Wiley Halsted*, *New York*, *and Wiley Eastern*, *New Delhi*, 1975)

B.D. Sharma and D.P. Mittal, *J. Math. Sci.* 10, 28 (1975) [unification of both previous entropic forms]

A. Wehrl, *Rev. Mod. Phys.* 50, 221 (1978)

q − GAUSSIANS (also called *kappa - distributions* or *generalized Lorentzians*):

Gaussian distribution	Abraham de Moivre (1733)
	Pierre Simon de Laplace (1774)
	Robert Adrain (1808)
	Carl Friedrich Gauss (1809)
Cauchy-Lorentz-Breit-Wigner distribution	Agustin Louis Cauchy (~1821)
	Hendric Antoon Lorentz (~1880)
Student's t-distribution	William Sealy Gosset (1908)

PHYSICAL REVIEW E **78**, 021102 (2008)

Nonadditive entropy reconciles the area law in quantum systems with classical thermodynamics

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(Received 16 March 2008; revised manuscript received 16 May 2008; published 5 August 2008)

The Boltzmann–Gibbs–von Neumann entropy of a large part (of linear size L) of some (much larger) d-dimensional quantum systems follows the so-called area law (as for black holes), i.e., it is proportional to L^{d-1}. Here we show, for $d=1,2$, that the (nonadditive) entropy S_q satisfies, for a special value of $q \neq 1$, the classical thermodynamical prescription for the entropy to be extensive, i.e., $S_q \propto L^d$. Therefore, we reconcile with classical thermodynamics the area law widespread in quantum systems. Recently, a similar behavior was exhibited in mathematical models with scale-invariant correlations [C. Tsallis, M. Gell-Mann, and Y. Sato, Proc. Natl. Acad. Sci. U.S.A. **102** 15377 (2005)]. Finally, we find that the system critical features are marked by a maximum of the special entropic index q.

SPIN ½ XY FERROMAGNET WITH TRANSVERSE MAGNETIC FIELD:

$$\hat{\mathcal{H}} = -\sum_{j=1}^{N-1} \left[(1+\gamma)\hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + (1-\gamma)\hat{\sigma}_j^y \hat{\sigma}_{j+1}^y + 2\lambda \hat{\sigma}_j^z \right]$$

$|\gamma| = 1$ → *Ising ferromagnet*

$0 < |\gamma| < 1$ → *anisotropic XY ferromagnet*

$\gamma = 0$ → *isotropic XY ferromagnet*

$\lambda \equiv$ *transverse magnetic field*

$L \equiv$ *length of a block within a* $N \to \infty$ *chain*

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

$\rho_N \equiv$ ground state $(T = 0)$ of the N-system

(assuming $\lambda^{xy} = +0$)

$\Rightarrow \rho_N^2 = \rho_N \Rightarrow Tr\rho_N^2 = 1$

$\Rightarrow \rho_N$ is a pure state

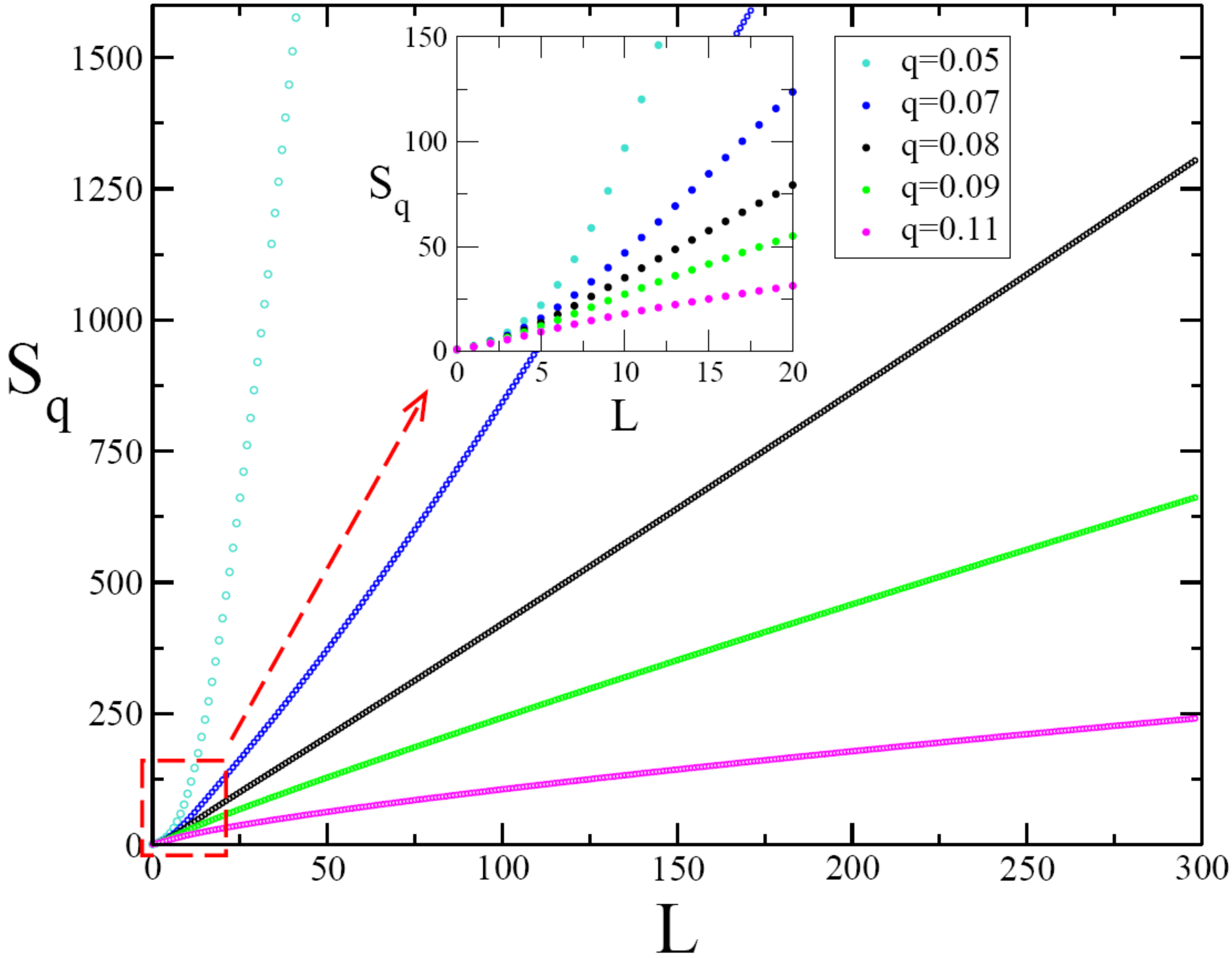
$\Rightarrow S_q(N) = 0 \quad (\forall q,\ \forall N)$

In contrast, $\rho_L \equiv Tr_{N-L}\ \rho_N$ satisfies $Tr\rho_L^2 < 1$

$\Rightarrow \rho_L$ is a mixed state

$\Rightarrow S_q(N, L) > 0$

ISING MODEL



F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Using a Quantum Field Theory result

*in P. Calabrese and J. Cardy, JSTAT P*06002 (2004)

we obtain, at the critical transverse magnetic field,

$$q_{ent} = \frac{\sqrt{9+c^2}-3}{c}$$

with $c \equiv$ *central charge in conformal field theory*

Hence

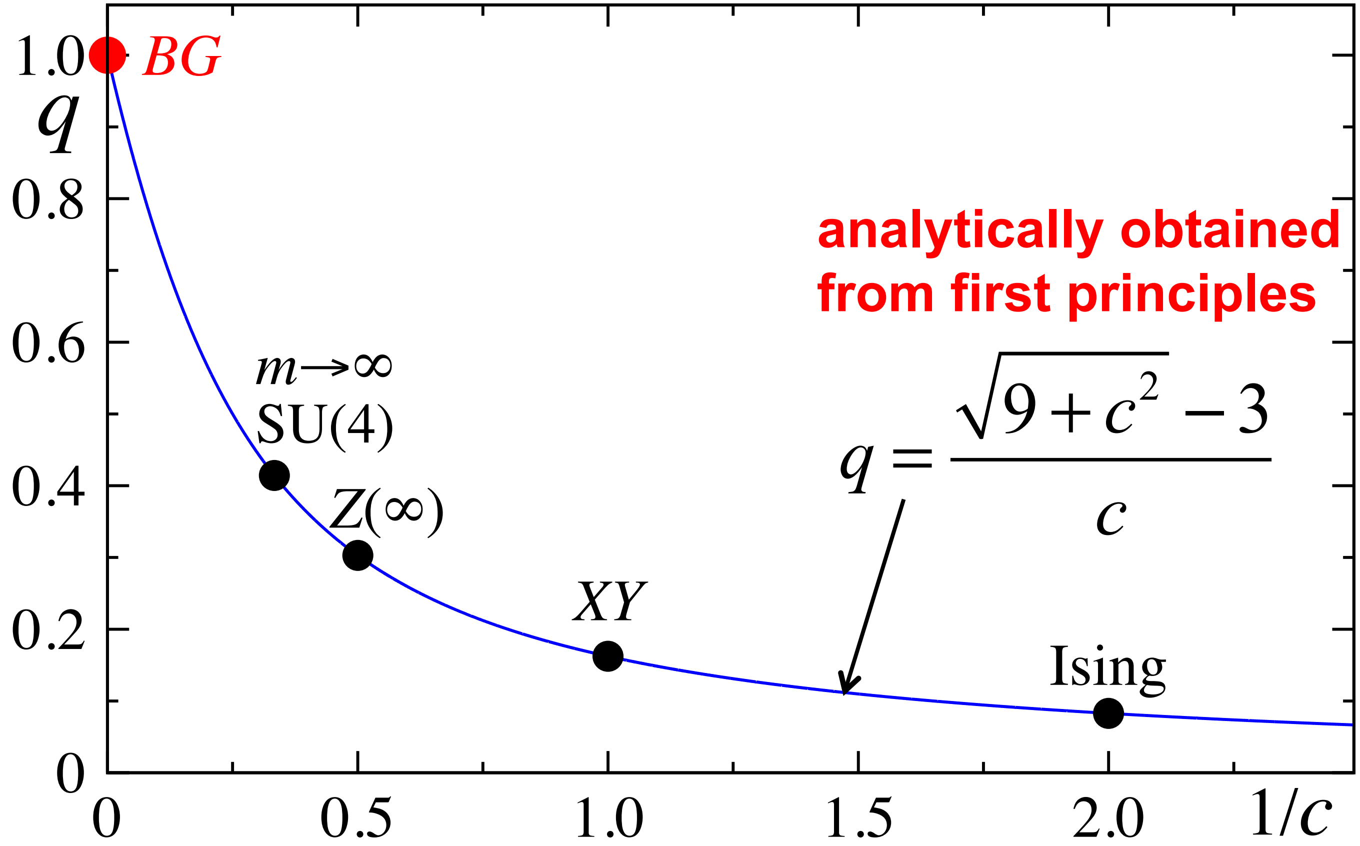
Ising and anisotropic XY ferromagnets $\Rightarrow \; c = \frac{1}{2} \;\Rightarrow\; q_{ent} = \sqrt{37}-6 \;\approx 0.0828$

and

Isotropic XY ferromagnet $\Rightarrow \; c = 1 \;\Rightarrow\; q_{ent} = \sqrt{10}-3 \;\approx 0.1623$

F. Caruso and C. T., Phys Rev E **78**, 021101 (2008)

Block entropy for the *d=1+1* model, with central charge *c*, at its quantum phase transition at *T=0* and critical transverse "magnetic" field



Self-dual $Z(n)$ magnet $(n=1,2,...)$ [FC Alcaraz, JPA 20 (1987) 2511]

$$\rightarrow c = \frac{2(n-1)}{n+2} \in [0,2]$$

$SU(n)$ magnets $(n=1,2,...;\ m=2,3,...)$ [FC Alcaraz and MJ Martins, JPA 23 (1990) L1079]

$$\rightarrow c = (n-1)\left[1 - \frac{n(n+1)}{(m+n-2)(m+n-1)}\right] \in [0,n-1]$$

BE CAREFUL!!!

For $d=1$ quantum system with central charge c, we have

$$S_{BG}=\frac{c}{3}\ln L+\ln b+\ldots=\ln\left(bL^{c/3}\right)+\ldots$$

and the extensive entropy is S_q with $q=\frac{\sqrt{9+c^2}-3}{c}$

But if we (wrongly) assume equal probabilities, we will (wrongly) use $S_{BG}=\ln W$, hence $W\sim bL^{c/3}$.

This corresponds to the power-law class, whose extensive entropy is S_q with $q=1-\frac{3}{c}$, which is definitively wrong!!!

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An IOP and SISSA journal

PAPER: Quantum statistical physics, condensed matter, integrable systems

Generalized isotropic Lipkin–Meshkov–Glick models: ground state entanglement and quantum entropies

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$$0 < \lim_{L\to\infty} \lim_{N\to\infty} \frac{S_q(L,N)}{L} < \infty$$

i.e.,

$$S_q(L,N) \propto L \qquad (N >> L >> 1)$$

$N \equiv$ total number of particles of the $d = 1$ system

$L \equiv$ total number of particles of the subsystem

$$q = 1 - \frac{2}{m-k} \qquad (m = 1,2,...;\ k = 0,1,2,...;\ m-k \geq 3)$$

$m \equiv$ number of internal degrees of freedom per particle

(e.g., $m = 2s$; $s \equiv$ spin size=$1/2$, 1, $3/2$,...)

$k \equiv$ number of fundamental-state vanishing magnons

Carrasco, Finkel, Gonzalez-Lopez, Rodriguez and Tempesta, JSTAT (2016)

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Nonadditive entropy for random quantum spin-S chains

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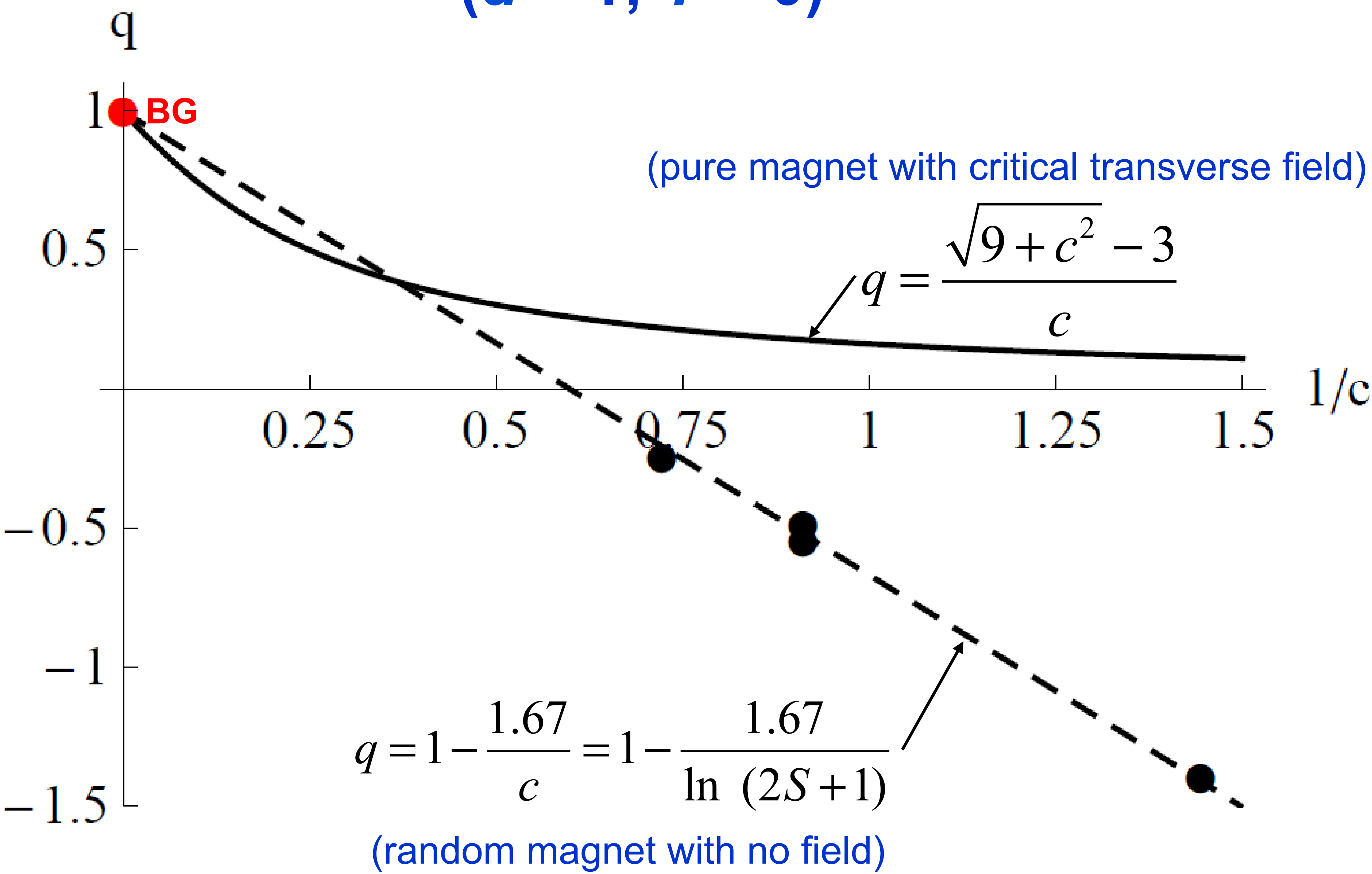
ABSTRACT

We investigate the scaling of Tsallis entropy in disordered quantum spin-S chains. We show that an extensive scaling occurs for specific values of the entropic index. Those values depend only on the magnitude S of the spins, being directly related with the effective central charge associated with the model.

$$H_{Heis} = \sum_{i=1}^{N} J_i \vec{S}_i \cdot \vec{S}_{i+1}$$

where $\{J_i\}$ are random exchange couplings obeying a probability distribution $P(J)$ and $\{\vec{S}_i\}$ are spin-S operators, with periodic boundary conditions

(d = 1; T = 0)



A Saguia and MS Sarandy, Phys Lett A **374**, 3384 (2010)

q – PRODUCT:

L. Nivanen, A. Le Mehaute and Q.A. Wang, Rep. Math. Phys. **52**, 437 (2003)
E.P. Borges, Physica A **340**, 95 (2004)

The q - product is defined as follows:

$$x \otimes_q y \equiv \left[x^{1-q} + y^{1-q} - 1 \right]^{\frac{1}{1-q}}$$

Properties :

$i)\quad x \otimes_1 y = x\, y$

$ii)\quad \ln_q (x \otimes_q y) = \ln_q x + \ln_q y$ (extensivity of Sq)

$[whereas\ \ \ln_q (x\, y) = \ln_q x + \ln_q y + (1-q)(\ln_q x)(\ln_q y)]$

(nonadditivity of Sq)